Avengers
College

UNIVERSITY OF
CAMBRIDGE
Computer Laboratory

Computer Science Admissions Test

EXAMPLE QUESTIONS. *These are meant as practice material and have various degrees of difficulty. Some are inspired from past CSAT papers.*

Instructions:

- The test duration is 100 minutes. Section A has 8 questions. Section B has 12 questions which are more challenging and worth more marks.
- All questions attempted are marked. Your best 5 questions from each section are considered. Partial answers are taken into account. You can choose the questions to answer and their order.
- Write only on the work-booklet provided and clearly label the question you are solving at the top of each page. Answers without working may not gain full marks. You should show sufficient working to make your solutions clear to the Examiner, but these need not be extremely thorough.
- Calculators, phones, watches, smart-glasses or other electronic devices or paper are **not** permitted.
- All paper must be handed in. Do not write on the cover or question sheets.
- **Do not** discuss any test questions with others (e.g. candidates at the same or another College, the Internet, or elsewhere), especially before March. You would disadvantage yourself.

It is recommended that you:

- take 5 minutes first to read through all questions,

- start with Section A and spend no more than 30 minutes on it,

- aim for 5 questions in each section; if you finish early then attempt more from Section B.

Good luck!

Section A — aim for 5 questions out of 8, of your choosing

1. You have a card of 10cm by 10cm. What is the largest volume in cm^3 of a box (without a lid) that can be obtained by cutting out a square of side x from each corner and then folding the flaps up?

2. Let $f(x)$ mean that function f is applied to x, and $f^n(x)$ mean $f(f(...f(x)))$, that is f is applied to x, n times. Let $g(x) = x + 1$ and $h_n(x) = g^n(x)$. What is $h_n^m(0)$?

3. Triangle ABC is isosceles with $AB = AC$. Let the circle having diameter AB and centre O intersect BC at some point P. Find the ratio BP/BC.

4. What is the units digit of the number $\sum_{n=1}^{1337}(n!)^4$?

5. Using only the functions $\max$ and $\min$ and arithmetic operations (no *if* clauses), express the amount of possible overlap between two intervals $[a_1, a_2]$ and $[b_1, b_2]$, where a_1, a_2, b_1, b_2 are arbitrary real numbers with $a_1 < a_2$ and $b_1 < b_2$.

6. Which values of k give a maximum at $x = -1$ for $f(x) = (k+1)x^4 - (3k+2)x^2 - 2kx$?

7. The Taylor expansion of $\ln(1 + x)$ is defined as $\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$. Expand $\ln\left(\frac{1-x}{1+x^2}\right)$ up to and including the 4th power of x.

8. Given a grid of 4×4 points, how many triangles with their vertices on the grid can be drawn?

Section B — aim for 5 questions out of 12, of your choosing

9. Player A rolls one die. Player B rolls two dice. If A rolls a number greater or equal to the largest number rolled by B, then A wins, otherwise B wins. What is the probability that B wins?

10. A circle of radius r is tangent at two points on the parabola $y = x^2$ such that the angle between the two radii at the tangent points is 2θ, where $0 < 2\theta < \pi$. Find r as a function of θ.

11. An organism is born on day $k = 1$ with 1 cells. During day $k = 2, 3, \ldots$ the organism produces $\frac{k^2}{k-1}$ times more new cells than it produced on day $k - 1$. Give a simplified expression for the total of all its cells after n days. *Hint:* This is different to the number of new cells produced during day n.

12. Let $n < 10$ be a non-negative integer. How many integers from 0 to 999 inclusive have the sum of their digits equal to n? Give your answer in terms of n. *Hint:* Try first for integers from 0 to 99.

13. On a grid of $m \times n$ squares, how many ways exist to get from the top-left corner to the bottom-right corner if you can only move right or down on an edge?

14. You must slice a square of side length b into 6 pieces with equal areas, using 3 lines that intersect each other inside the square, one of which is a diagonal. Where on the sides of the square, with respect to the nearest corner, should the other two lines cut? Give your answer in terms of b.

15. Does 30 divide $n^5 - n$ for all positive integers n?

16. Dividing x by a small annual r-percent cumulative interest rate approximates the number of years needed to double your investment with a bank. Find x. *Hint:* The definition from Q7 may come in handy, and the keyword "small" may be important.

17. Given a sequence of zeros and ones of length n, let L_n be the number of sequences that have no adjacent zeros. Give a recursive formula for L_n.

18. You travel on Earth's surface south n miles, then east n miles, then north n miles and find yourself back where you started, without visiting any point more than twice. What is the closest you could have been to the south pole when you started? Assume Earth is a sphere with radius $R > n$.

19. Let f be the recursive function below, with $f(1) = f(0) = 0$. What is the value of $f(1337)$?

$$\begin{array}{ll} f(n) = f(n-1) - 1 & \text{if } n \text{ is divisible by 2 or 3} \\ f(n) = f(n-2) + 2 & \text{otherwise} \end{array}$$

20. Evaluate $\lim_{n\to\infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}\right)$. *Hint:* Graph sketching can help, and you may use the relation $\lim_{n\to\infty} f\left(\frac{an+b}{cn+d}\right) = f\left(\frac{a}{c}\right)$ when f is continuous and a, b, c, d are reals with $c \neq 0$.

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